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PERTURBATIVE QCD AND ELECTROMAGNETIC FORM FACTORS

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ABSTRACT

We calculate nucleon magnetic form factors using perturbative QCD for several distribution amplitudes including a general one given in terms of Appell polynomials. We find that the magnitude and sign of both nucleon magnetic form factors can be explained within perturbative QCD. The observed normalization of G_{Mp} requires that the distribution amplitude be broader than its superhigh momentum transfer limit, and the G_{Mn}/G_{Mp} data may require the distribution amplitude to be asymmetric, in accord with distribution amplitudes derived from QCD sum rules. Some speculation as to how an asymmetric distribution amplitude can come about is offered. Finally, we show that the soft contributions corresponding to the particular distribution amplitudes we use need not be bigger than the data.

I. INTRODUCTION

There has been much discussion about the validity of using perturbation theory with QCD (PQCD) to make predictions for

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exclusive processes at experimentally feasible energies.^{1,2,3,4} It is generally granted that the predicted scaling behaviour works, by luck or otherwise, at reasonably low Q^2 . Absolute normalisations then become the next testing ground for PQCD. Unfortunately the normalization, unlike the energy or momentum transfer scaling behavior, is dependent upon unknown and/or perhaps not well understood wave functions of the quarks in a hadron.^{1,2,5,6} Still, the question remains whether wave functions can be found for which the calculated normalizations are in agreement with the data. In this paper it is shown that such wave functions can be found, and the nature of these wave functions is examined, but no firm stand can be taken on whether or not these wave functions are the correct ones. Ab initio calculations of the correct nuclear wave functions requires the use of non-perturbative techniques beyond the scope of this paper.

In order to clarify the discussion, several categories of predictions of PQCD for exclusive processes can be distinguished.

(1) Scaling behavior -- high-energy or high-momentum transfer scaling behavior of form factors or differential cross sections can be obtained.⁷ Taking electromagnetic form factors as an example, the helicity conserving one is always the biggest and goes like

$$F(Q^2) = \frac{A}{(Q^2)^{N-1}} \left(1 + \frac{b}{Q^2} + \dots \right) \quad (1)$$

for a system of N constituents. Predictions of power law behavior tend to work well. Figure 1 shows one example: the proton magnetic form factor. The currently published data⁸ for $Q^4 G_{Mp}$ is shown as a function of Q^2 and the PQCD scaling behavior appears substantially right for $Q^2 \geq 5 \text{ GeV}^2$.

(2) Normalization -- the normalization of the form factors (the coefficient A in Eq. 1) or scattering amplitudes could be obtained. These calculations depend on the quark wave functions, and in this paper explicit calculations for the nucleon magnetic form factors are shown. (There has been a claim^{3,4} that no reasonable wave function can give a PQCD calculated form factor as large as the data, so that the agreement seen with the PQCD scaling behavior in Fig. 1 is just luck.)

(3) Logarithmic corrections -- the logarithmic corrections to the power law behavior can be calculated.^{1,2} Like the normalization, these calculations are in general wave function dependent. For example, the leading term in a form factor is more completely given as

$$F(Q^2) = \frac{1}{(Q^2)^{N-1}} \left[a_s(Q^2) \right]^{N-1} \sum_{ij} d_{ij} \left(\log Q^2/\Lambda^2 \right)^{-\gamma_i - \gamma_j} . \quad (2)$$

The γ_j are calculable, positive, and monotonically increasing with j , but the d_{ij} are wave function dependent. Only one prediction is wave function independent, and that is the $\log Q^2$ behavior at sufficient Q^2 that only one term in the above sum survives. This requires extremely high Q^2 . In contrast to category (1), "logarithmic asymptopia" is now needed rather than "power law asymptopia;" that is to say, $\log Q^2$ must be large rather than just Q^2 being large.

(4) Polarization -- quantities specifically involving polarization can be calculated.⁹

In this paper the normalization of the nucleon magnetic form factor, category (2) above, will be studied, using plausible wave functions or distribution amplitudes including a flexible class of distribution amplitudes that can be expressed in terms of the first six Appell polynomials.^{1,2,6} (These are eigensolutions of the evolution equation for the distribution amplitude. For this paper any basis set would do as well, but the Appell polynomials are very convenient for any studies of the logarithmic Q^2 dependence of the form factor.) It will be shown that it is possible to match at high Q^2 the observed normalization of G_{Mp} , without running afoul of wave function normalization conditions. The simplest wave functions, however, are not the best ones to use. The observed normalization of G_{Mp} requires a broad distribution amplitude, and moreover the observed value of the ratio G_{Mp}/G_{Mn} may not be readily explained without asymmetric distribution amplitudes.⁶ Indeed, a distribution amplitude based on QCD sum rules which shows an asymmetry in the quark spatial wave function has already been suggested by Chernyak and Zhitnitsky⁵. The foregoing is discussed in Section II. Section

IV includes some speculation about how it is possible to have an asymmetric distribution amplitude even though $SU(6)$ results, which are based on completely symmetrical spatial wave functions, work fairly well for quantities measured at low Q^2 .

Section III contains a study of the "soft contributions."⁴ In QCD the impulse approximation gives the correct leading order (in $1/Q^2$) form factors at high Q^2 and the result comes from the tail, meaning the high transverse momentum part, of the quark wave function. High enough transverse momenta allow using perturbation theory with QCD and indeed the PQCD calculation can be seen as a way of generating the correct tail of the wave function and immediately using it in an impulse approximation calculation of the form factor. If we do an impulse approximation calculation of the form factor keeping only the low transverse momentum part of the wave function, for example by purposely using a wave function like a gaussian which is plausible at low momenta but falls much too quickly at high momenta, then we get the "soft contributions." At high Q^2 the soft contributions fall faster than the PQCD or "hard" contributions so the PQCD result must eventually dominate, but at any Q^2 the size of the contributions is a test of the validity of the approximations that go into the PQCD result. The question is whether the soft contributions are significant or even dominant at present experimental Q^2 . It is found for some wave functions of interest that the soft contributions may be important but are not necessarily larger than the PQCD result. As discussed in Section III, it is important to note the effects of the tail of the wave function on wave function parameters.

A summary and some speculation is given in Section IV.

II. NUCLEON FORM FACTORS IN PQCD

One can show that in QCD the impulse approximation contains the leading contribution to the form factor at high Q^2 and further that the leading contribution to either F_1 or G_M (these two are equivalent to leading order since F_2 falls faster by one power of Q^2) is [Fig. (2)]

$$G_M = \int [dx] [dy] \phi^*(x, Q) T_H(x, y, Q) \phi(y, Q) . \quad (3)$$

where ϕ , the three quark distribution amplitude, and the other quantities in Eq. (3) are defined below. To obtain the leading contribution it is sufficient to consider the three quark part of the wave function; Fock components with more constituents require more gluon exchanges to give all the constituents parallel momenta in the final state and their contribution to the form factor fall faster with Q^2 . We work in an infinite momentum frame where the entering proton is moving along the z-axis. The transverse momentum components of the i th quark are

$$k_{iT} = (k_i^1, k_i^2) \quad (4)$$

and the momentum fractions are

$$x_i = k_i^+ / p^+ \quad (5)$$

where $k_i^+ = k_i^0 + k_i^3$ and p is the proton momentum. For the three quarks in the initial proton

$$\sum k_{iT} = 0 \quad (6)$$

and

$$\sum x_i = 1. \quad (7)$$

In the expression for the form factor, the distribution amplitude ϕ is

$$\phi(x, Q) = \int^Q [dk_T] \psi(x, k_T) \quad (8)$$

where ψ is the three quark wave function. The differentials are

$$[dx] = \prod dx_i \delta(1 - \sum x_j) \quad (9)$$

and

$$[dk_T] = \Pi \left[\frac{d^2 k_{iT}}{16\pi^3} \right] \cdot 16\pi^3 \delta^{(2)}(\Sigma k_{iT}) \quad (10)$$

The wave function is normalized by

$$\int [dx] [dk_T] |\psi(x, k_T)|^2 = P_{3q} \quad (11)$$

The "hard scattering amplitude" T_H is the scattering amplitude for three parallel quarks going into three parallel quarks. There are 42 diagrams that can be drawn for T_H , but only 14 are non-zero and only the four drawn in Fig. 3 need be calculated, the others being obtained by symmetries. If e_j is the operator that gives the charge of quark j , then^{1,2}

$$T_H = \left(\frac{8\pi C_B a_s(Q^2)}{Q^2} \right)^2 \sum_{j=1}^3 \{e_j T_j + (x \leftrightarrow y)\} \quad (12)$$

with $C_B = 2/3$, and

$$\begin{aligned} T_1 &= \frac{1}{x_3(1-x_1)^2} \frac{1}{y_3(1-y_1)^2} + \frac{1}{x_2(1-x_1)^2} \frac{1}{y_2(1-y_1)^2} \\ &\quad - \frac{1}{x_2 x_3(1-x_3)} \frac{1}{y_2 y_3(1-y_1)} = T_3(1 \leftrightarrow 3) \end{aligned} \quad (13a)$$

and

$$T_2 = \frac{1}{x_1 x_3(1-x_1)} \frac{1}{y_1 y_3(1-y_3)} \quad (13b)$$

Note the $1/Q^4$, the $\log(Q^2/\Lambda^2)$ dependence within the strong coupling parameters a_s , and the singularities near the kinematic boundaries of x_i and y_i [$0 \leq x_i, y_i \leq 1$].

The proton wave function is not at present calculable. However, QCD sum rules allow some moments of the proton wave function to be

determined, which in turn sets conditions on model wave functions,⁵ and lattice gauge theories may eventually give an ab initio calculation. Some progress is being made on the pion wave function using lattice gauge theory.¹⁰

Accordingly G_M will be calculated for two general classes of wave functions, starting with a simple one-parameter symmetric wave function. This will show what is necessary to get the right normalization and will also display the limitations of symmetric spatial wave functions that seem to persist even in more sophisticated versions of the same.

(A) Simple symmetric wave function. A simple and factorable form of the wave function is⁴

$$\phi(x, k_T) = N(x_1 x_2 x_3)^\eta e^{-\frac{1}{2} k_{iT}^2 / a^2} \quad (14)$$

from which

$$\phi(x) = N' (x_1 x_2 x_3)^\eta \quad (15)$$

This is a one-parameter family of wave functions, the parameter being the power η . The constants N and N' are fixed by the wave function normalization condition, and the parameter a by the RMS value of k_T , which should be some reasonable value; one value suggested¹¹ is $a = 0.32$ GeV. A gaussian in the transverse momentum is incorrect at high k_T and section III will show the effect of additional terms in the transverse momentum wave function. However, the gaussian is useful for now to show how the usual normalization condition on the wave function implies constraints between the size of G_{Mp} and the RMS quark transverse momenta.

The integrals for G_M can be done analytically. The power must satisfy $\eta > 1/2$ to make those integrals converge. One way to begin looking at the results is to examine the ratio G_{Mp}/G_{Mn} , plotted in

Fig. 4. The proton form factor has a zero at $\eta=1$ and the neutron form factor has a zero at

$$\eta = \frac{1}{2} + \frac{1}{\sqrt{12}} \approx 0.79 . \quad (16)$$

Of course, the neutron and proton form factors have opposite signs and this constrains the values of η which may be chosen. Further, since it is the proton form factor that is positive, η is constrained to $1/2 < \eta \leq 0.79$. (Incidentally, there was for a while an error in the literature in the overall sign of T_H and hence of the calculated G_{Mp} . This had the effect of requiring $\eta > 1$ with consequent large effect on the normalization.)

The result for G_{Mp} using the wave function (14) is

$$Q^4 G_{Mp}(Q^2) = \frac{1}{\pi^2} a_s^2 a^4 \frac{64}{27} \frac{1-\eta}{\eta} N_\eta^2 X^2 , \quad (17)$$

with

$$N_\eta^2 = \frac{(6\eta+2)!}{[(2\eta)!]^3} \quad (18)$$

and

$$X = \frac{(\eta!)^2 (\eta-1)! (2\eta-2)!}{(2\eta)! (3\eta-1)!} \quad (19)$$

For $\eta=0.6$, $a_s = 0.3$, and $a = 0.32$ GeV this gives

$$Q^4 G_{Mp} \approx 1.1 \text{ GeV}^4 \quad (20)$$

which is near the data for $Q^2 \geq 5 \text{ GeV}^2$.

This wave function demonstrates that the observed size of the proton form factor can be obtained, at least at high Q^2 , from PQCD. It shows that the experimental normalization requires a broad ($\eta < 1$) rather than a narrow ($\eta > 1$) distribution amplitude. It also shows the possibility of a catastrophe that did not occur: it could have happened that the distribution amplitudes that gave a

normalization compatible with experiment also gave the wrong sign for G_{Mp} , but this was not the case.

On the other hand, this wave function may be too simple, and whether it can give the value of G_{Mn}/G_{Mp} seen experimentally depends delicately on just what that ratio is. Some comments on the G_{Mn} data are in order.

Strictly speaking, there is no G_{Mn} data away from $Q^2=0$. There is data on σ_n , the differential cross section for e-n elastic scattering, at $Q^2 = 2.5, 4, 6, 8$ and 10 GeV^2 but only at one angle.¹² Hence a separation of G_{En} and G_{Mn} is impossible. However, several things can be learned or are suggestive about G_{Mn} by analyzing the data that is available.

The salient observation is that σ_n/σ_p falls roughly like $1/Q^2$ from (say) 5 GeV^2 to 10 GeV^2 and is roughly $1/4$ at the upper Q^2 . Neglecting G_E gives $\sqrt{\sigma_n/\sigma_p} \approx G_{Mn}/G_{Mp}$ and since the contribution of G_{Ep} to σ_p really appears negligible at high Q^2 , we can safely say

$$\left| \frac{G_{Mn}}{G_{Mp}} \right| \leq \frac{1}{2} ,$$

at $Q^2 = 10 \text{ GeV}^2$. (The sign is known to be negative at $Q^2 = 0$.)

The falling cross section ratio suggests more. Possibly, σ_n/σ_p falls because G_{Mn} is dominating σ_n but $Q^4 G_{Mn}$ is not yet constant. However, the observed approximate constancy of $Q^4 G_{Mp}$ makes an alternative explanation plausible. Possibly, G_{Mn} is small, i.e., its leading term at high Q^2 is small, and then the cross section σ_n is dominated by G_{En} . This requires G_{En} to be about the same size as F_{1p} or G_{Mp} and leads naturally to $\sigma_n/\sigma_p \sim 1/Q^2$. Incidentally, since the leading term of G_{En} and F_{1n} are the same, one can tie the high and low Q^2 data together with the suggestion that $F_{1n} \approx 0$ at all Q^2 .

Returning to the simple symmetric wave function, we can consider three possibilities.

- (i) $G_{Mn}/G_{Mp} \approx -1/2$. More precisely, consider that G_{Mn}/G_{Mp} is falling with Q^2 until $Q^2 = 10 \text{ GeV}^2$ but is approximately constant at the value $(-1/2)$ thereafter. The simple symmetric wave function cannot give this value; values of

$|G_{Mn}/G_{Mp}|$ between $(-1/3)$ and (-1) are inaccessible (see Fig. 4).

- (ii) G_{Mn}/G_{Mp} small. This possibility means the distribution amplitude gives small G_{Mn}/G_{Mp} at high but experimentally accessible Q^2 . [If we let Q^2 be superhigh, then the evolution with changing $\log Q^2$ of the distribution amplitude must be taken into account and the ultimate consequence of this is known to give $Q^4 G_{Mp} \rightarrow 0$. On the other hand, the same superhigh Q^2 limit gives G_{Mn} positive, so that G_{Mn} must have a zero at some finite though possibly superhigh Q^2 and the ratio G_{Mn}/G_{Mp} should fall to zero before it ultimately becomes infinite.] This possibility would require $\eta \approx 0.79$ in the simple symmetric distribution amplitude, but this has the high price of requiring $\alpha = 0.67$ GeV (for $\alpha_s = 0.3$) to give the observed normalization of $Q^4 G_{Mp}$.
- (iii) G_{Mn}/G_{Mp} intermediate. This means G_{Mn}/G_{Mp} about $(-1/3)$ to $(-1/4)$. G_{Mn} is small enough that σ_n is still dominated by G_{En} , so the $1/Q^2$ fall-off of σ_n/σ_p is still naturally explained. This value of G_{Mn} gives no problem for the simple symmetric distribution amplitude. The example $\eta = 0.6$ fits here.

(B) More general wave function. For a more general treatment, expand $\phi(x)$ in polynomials (times a weight factor $x_1 x_2 x_3$ to decrease sensitivity to the end point singularities¹³). For the present purposes the particular choice of polynomials is not crucial, but it is convenient to use polynomials which are standard^{1,2} to the subject and which would be useful for study of logarithmic dependencies. The wave equation for $\phi(x, k_T)$ has a kernel which is dominated by one gluon exchange for high k_T . This observation can be turned into an "evolution equation" that governs how the distribution amplitude $\phi(x, Q^2)$ changes with Q^2 for high Q^2 . The evolution

equation can be solved by the separation of variables to yield

$$\phi(x, Q^2) = x_1 x_2 x_3 \sum N_i \gamma_i(x) \quad (21)$$

where

$$N_i = N_i(Q^2) = n_i \log^{-\gamma_i} \left(\frac{Q^2}{\Lambda^2} \right) \quad (22)$$

The n_i are non-calculable constants, but the γ_i are calculable^{1,2} and are positive and monotonically increasing with i . The first six "Appell polynomials" are

$$\begin{aligned} \gamma_0 &= 1 \\ \gamma_1 &= x_1 - x_3 \\ \gamma_2 &= 2 - 3(x_1 + x_3) \\ \gamma_3 &= 2 - 7(x_1 + x_3) + 8(x_1^2 + x_3^2) + 4x_1 x_3 \\ \gamma_4 &= x_1 - x_3 - \frac{4}{3}(x_1^2 - x_3^2) \\ \gamma_5 &= 2 - 7(x_1 + x_3) + \frac{14}{3}(x_1 + x_3)^2 + 14x_1 x_3 \end{aligned} \quad (23)$$

Quarks 1 and 3 are the ones with parallel spin and some of the above are symmetric and some antisymmetric under interchange of quarks 1 and 3. If ϕ is split into parts ϕ_S and ϕ_A that are symmetric and antisymmetric under $1 \leftrightarrow 3$, then ϕ_S and ϕ_A can be associated with the corresponding symmetry spin-isospin wave functions for the proton and neutron

$$\begin{aligned} \phi_p(x) &= \phi_S(x) (2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow) / \sqrt{6} \\ &+ \phi_A(x) (u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow) / \sqrt{2} + \text{perm} \end{aligned} \quad (24a)$$

and

$$\begin{aligned} \phi_n(x) = & \phi_S(x) (d_\uparrow d_\downarrow u_\uparrow + u_\uparrow d_\downarrow d_\uparrow - 2d_\uparrow u_\downarrow d_\uparrow) / \sqrt{6} \\ & + \phi_A(x) (u_\uparrow d_\downarrow d_\uparrow - d_\uparrow d_\downarrow u_\uparrow) / \sqrt{2} + \text{perm} . \end{aligned} \quad (24b)$$

It is now straightforward to calculate the form factors and normalization condition. The results are^{14,15}

$$\begin{aligned} Q^4 G_{Mp}(Q^2) = & \left(\frac{4\pi a_s}{27} \right)^2 \{ 20 N_1^2 - 42\sqrt{3} N_0 N_1 \\ & + 36 N_2^2 + 28\sqrt{3} N_1 N_2 - 54 N_0 N_2 \\ & + 188 N_3^2 - 144 N_2 N_3 - \frac{220}{\sqrt{3}} N_1 N_3 + 198 N_0 N_3 \\ & + \frac{26}{27} N_4^2 + \frac{46}{\sqrt{3}} N_3 N_4 - \frac{22}{\sqrt{3}} N_2 N_4 - \frac{26}{3} N_1 N_4 + 10\sqrt{3} N_0 N_4 \\ & + \frac{77}{9} N_5^2 - \frac{41}{9\sqrt{3}} N_4 N_5 - 59 N_3 N_5 + 11 N_2 N_5 + \frac{35}{\sqrt{3}} N_1 N_5 - 42 N_0 N_5 \} \end{aligned} \quad (25a)$$

$$Q^4 G_{Mn}(Q^2) = \left(\frac{4\pi a_s}{27} \right)^2 \{ 54 N_0^2 \quad (25b)$$

$$\begin{aligned} & + 22 N_1^2 + 42\sqrt{3} N_0 N_1 \\ & - 6 N_2^2 - 28\sqrt{3} N_1 N_2 + 54 N_0 N_2 \\ & - \frac{170}{3} N_3^2 + 60 N_2 N_3 + \frac{220}{\sqrt{3}} N_1 N_3 - 30 N_0 N_3 \\ & + \frac{26}{27} N_4^2 - \frac{46}{\sqrt{3}} N_3 N_4 + \frac{22}{\sqrt{3}} N_2 N_4 - \frac{26}{3} N_1 N_4 - 10\sqrt{3} N_0 N_4 \\ & - \frac{145}{54} N_5^2 + \frac{41}{9\sqrt{3}} N_4 N_5 + \frac{65}{3} N_3 N_5 - \frac{5}{3} N_2 N_5 - \frac{35}{\sqrt{3}} N_1 N_5 + 20 N_0 N_5 \} \end{aligned}$$

The G_M have now been given for a 6 parameter family of wave functions. Note that there is no N_0^2 term for the proton. This is the same as the zero at $\eta=1$ in the previous wave function.

The N_i cannot be made arbitrarily large because there is a wave function normalization condition to satisfy. Some trade-off between normalization of the distribution amplitude and normalization of the transverse momentum part of the wave function is possible, but large N_i will generally lead to large and possibly unacceptable RMS quark transverse momenta. It can be seen how this happens for a factorizable wave function with a gaussian k_T -dependence. Let

$$\phi(x, k_T) = \phi(x) g(k_T) \quad (26)$$

where

$$\int [dk_T] g(k_T) \equiv \langle g \rangle = 1 \quad (27)$$

and

$$g(k_T) = \frac{192\pi^4}{a^4} e^{-\mathbb{L}k_{iT}^2/2a^2} \quad (28)$$

The normalization condition becomes

$$\int [dx] \phi^2(x) \int [dk_T] g^2(k_T) \equiv \langle \phi^2 \rangle \langle g^2 \rangle = P_{3q} \quad (29)$$

or

$$\begin{aligned} & \frac{1}{165} \left[165 N_0^2 + 11 N_1^2 + 33 N_2^2 + 17 N_3^2 + \frac{1}{3} N_4^2 \right. \\ & + \frac{53}{9} N_5^2 - 44 N_0 N_3 + 2 N_2 N_3 + \frac{2}{3} N_1 N_4 - \frac{22}{3} N_0 N_5 \\ & \left. - \frac{14}{3} N_2 N_5 + \frac{2}{3} N_3 N_5 \right] = \frac{7!}{48\pi^4} a^4 P_{3q} , \end{aligned} \quad (30)$$

where P_{3q} is the 3 quark probability. If only one N_i is non-zero,

the most form factor for a fixed normalization occurs if $i = 3$. For this case with $P_{sq} = 1$ and $a_s = 0.3$, the proton form factor data at $Q^2 \geq 5 \text{ GeV}^2$ is fit with $a = 0.39 \text{ GeV}$, a reasonable value. Other choices of i would tend to give larger, less acceptable values of a .

Let us discuss requirements on the distribution amplitude if it is to give the observed G_{Mp} and G_{Mn} and not imply via the normalization condition a quark transverse momentum, measured by a , which is unacceptably big. The possibilities for the neutron data will match our earlier discussion, and all remarks will apply to Appell polynomial expansions through the quadratic Appell polynomials.

- (i) $G_{Mn}/G_{Mp} \approx -1/2$. Can we obtain this with only the symmetric Appell polynomials contributing to the distribution amplitude and with an acceptable a , say a below 700 MeV? The answer is no. (Our search routine is simple: we scan on a tight enough grid all the $\{N_i\}$ that will give a below 700 MeV, searching for sets of $\{N_i\}$ that give $Q^4 G_{Mp}$ in the range 0.9 to 1.1 GeV^4 and $Q^4 G_{Mn}$ in the range -0.4 to -0.6 GeV^4 , both with $a_s = 0.3$. There are no such sets.)

Using only anti-symmetric Appell polynomials is clearly futile, since they give $G_{Mn} > G_{Mp}$. A distribution amplitude that works quite well at giving both G_{Mp} and G_{Mn} is (for $P_{sq} = 1$, $a_s = 0.3$, and $a = 0.39 \text{ GeV}$).

$$\phi_M(x) = [0.38 \text{ GeV}^2] x_1 x_2 x_3 [\gamma_3 - \gamma_1] \quad (31)$$

This amplitude is perhaps surprising because it is asymmetric, and Section IV below contains some speculation on how this might come about.

- (ii) G_{Mn}/G_{Mp} small. Here we can succeed with symmetric Appell polynomials. The smallest a we can find for $Q^4 G_{Mp} = 1.0 \text{ GeV}^4$ and $Q^4 G_{Mn}$ between $\pm 0.1 \text{ GeV}^4$ is $a = 0.48 \text{ GeV}$ (for the record, $N_0 = 0.2$, $N_2 = 0.2$, $N_3 = 0.57$, $N_5 = 0.5$, all in GeV^2 , and $Q^4 G_{Mn} = -0.07 \text{ GeV}^4$). This a however is only

borderline acceptable as it gives a somewhat large $\langle k_T \rangle$ for the quarks and the difficulties of escaping some problems associated with the "soft contributions" (see section III) go like a^4 or higher.

- (iii) G_{Mn}/G_{Mp} intermediate. As there was no problem with the simple symmetric distribution amplitude, there is none here. The smallest a for the symmetric case and $Q^4 G_{Mp} = 1.0 \text{ GeV}^4$ is here and is $a = 0.37 \text{ GeV}$. (Again for the record, $N_0 = 0.1$, $N_2 = -0.1$, $N_3 = 0.43$, $N_5 = 0.0$, all in GeV^2 , and $Q^4 G_{Mn} = -0.28 \text{ GeV}^4$.)

For now it should be emphasized that the apparent asymptotic G_M has a size as well as a Q^2 fall-off that can be matched in PQCD with reasonable values of the QCD coupling constant and quark transverse momenta.

III. SOFT CONTRIBUTIONS

The PQCD expression for the form factor can be derived as an approximation to the impulse approximation. At high Q^2 only the "tail" or high k_T part of the wave function is important and this is the piece that can be calculated in PQCD and substituted into the usual impulse approximation to obtain the PQCD result for G_M . Note that while the impulse approximation is the dominant contribution¹ to the form factor at high Q^2 , the same is not necessarily true at low Q^2 .

There remain low k_T parts of the wave function that can make contributions to G_M . These are the "soft contributions"⁴ and are not included in PQCD. It is (or will be) clear enough that they fall faster with Q^2 than the PQCD or "hard" contributions. How big are they at Q^2 's where experiments are done? For the wave functions defined by Eqs. (21) and (26) the answer is that they are big--unfortunately or fortunately--but modifications can be made in the k_T dependence to make them small. This will be discussed in this section.

The impulse approximation in the infinite momentum frame formalism can be written in the following symmetric form

$$Q^4 G(Q^2) = Q^4 \int_1 [dx] [d^2 k_T] \phi(x, h_T^-) e_i \phi^*(x, h_T^+) \quad (32)$$

where ϕ includes the spin-isospin part of the wave function, h_T^\pm are the transverse momenta of the quarks in the case where the first quark is struck (see Fig. 5)

$$\begin{aligned} h_{1T}^\pm &= k_{1T} \pm \frac{1}{2}(1-x_1)q \\ h_{2T}^\pm &= k_{2T} \pm \frac{1}{2} x_2 q \\ h_{3T}^\pm &= k_{3T} \pm \frac{1}{2} x_3 q \end{aligned} \quad (33)$$

and e_i is the charge of the struck quark. For simplicity, Eq. (32) will be evaluated only for symmetric quark wave functions in what follows, and the factored form Eq. (26) will be used with $g(k_T)$ initially taken to be given by Eq. (28). In this case, the impulse approximation reduces to

$$\begin{aligned} Q^4 G(Q^2) &= 4\pi^4 \frac{Q^4}{a^4} \int [dx] \phi_S^2(x) \exp\left[-(x_2^2 + x_3^2 + x_2 x_3) Q^2 / 2a^2\right] \\ &= f\left(\frac{Q^2}{2a^2}\right) \end{aligned} \quad (34)$$

where the function $f(\xi)$ is shown in Fig. 6 for the case when

$$\phi_S = \phi_3 = x_1 x_2 x_3 N_3 \gamma_3(x), \quad (35)$$

and with $N_3^2 = 0.3$ as required if ϕ_3 is to fit the asymptotic proton data. Note that $f(\xi)$ peaks at $\xi \simeq 100$, which corresponds to $Q \simeq 30(\text{GeV}/c)^2$ for $a = 0.39 \text{ GeV}$, and that it has a maximum value of about $3 (\text{GeV})^4$, about three times larger than the experimental value of $1 (\text{GeV})^4$. Furthermore, $f(\xi)$ does not fall to the experimental value until Q^2 is greater than $300 (\text{GeV}/c)^2$. This shows that the

low k_T components of this wave function are dominating over the high k_T parts; to make (34) smaller than the hard calculation (25) for $Q^2 \geq 10 \text{ (GeV/c)}^2$ would require that a^2 be 30 times smaller, which would in turn violate the normalization condition (30). The result may appear surprising, since the Gaussian wave function (28) has an RMS transverse momenta of only $\sqrt{2} a$, and falls off rapidly with k_T .

However, it is not difficult to find a wave function $g(k_T)$ which will give a smaller result for the impulse approximation, and will at the same time be consistent with the hard scattering calculation and the normalization condition. Let

$$g(k_T) = 3(16\pi^2)^2 \left\{ \frac{A}{4a^4} e^{-[p_\rho^2 + p_\lambda^2]/2a^2} + \frac{B \theta}{[p_\rho^2 + \epsilon \kappa^2][p_\rho^2 + \epsilon p_{\lambda T}^2]} \right. \\ \left. = g_{\text{soft}}(k_T) + g_{\text{hard}}(k_T) \right. \quad (36)$$

where $p_\rho = (\underline{k}_{1T} - \underline{k}_{3T})/\sqrt{2}$ and $p_\lambda = (\underline{k}_{1T} + \underline{k}_{3T} - 2\underline{k}_{2T})/\sqrt{6}$.

Choosing $\epsilon \simeq 1/16$ will prove convenient, and $\theta \equiv \theta(p_\rho - \kappa)\theta(p_\lambda - \kappa)$ so that θ is 1 if both p_ρ and p_λ are above κ and otherwise is zero. The tail falls asymptotically like four powers of momentum as it should but otherwise is chosen only for purposes of illustration.

The linear normalization condition for g tells us that

$$A + B I = 1 \quad (37)$$

with

$$I = \frac{1}{\epsilon} \int_1^{Q^2/\kappa^2} \frac{dx}{x+\epsilon} \ln \frac{x+\epsilon Q^2/\kappa^2}{x+\epsilon} \quad (38)$$

We will evaluate I numerically (e.g., for $\epsilon = 1/16$, $Q^2 = 10 \text{ GeV}^2$, and $\kappa = 300 \text{ MeV}$ we get $I = 51.$), but note that

$$\lim_{Q^2 \rightarrow \infty} I(\epsilon, Q^2/\kappa^2) = \frac{1}{2\epsilon} \ln^2 \frac{Q^2}{\kappa^2}.$$

The usual normalisation condition takes the form (for the pure ϕ_s case)

$$\frac{A^2}{a^4} + \frac{8AB}{a^4} R + \frac{8B^2}{(1+\epsilon)^2 \kappa^4} = \frac{1}{48\pi^4 \langle \phi^2 \rangle} \simeq 57.2 \text{ GeV}^{-4} \quad (39)$$

with ϕ_s chosen so that the PQCD result gives 2/3 of the experimental result for $Q^4 G_{\text{mp}}$ at $Q^2 = 10 (\text{GeV}/c)^2$. (We will allow the soft contribution to give the remaining 1/3 -- see below.) The overlap term involves

$$R = \int_1^\infty dx dy \frac{e^{-(x+y)x^2/2a^2}}{(x+\epsilon)(x+\epsilon y)} \quad (40)$$

and R is easily bounded,

$$R < \frac{2a^2}{\kappa^2} e^{-\kappa^2/a^2} \leq \frac{2}{e} \simeq 0.74$$

This suffices to make the overlap term negligible for the A and B we will work with below.

At high Q^2 , a simple formula can be obtained for the impulse approximation (32), if we use the approximate relations

$$\int [d^2 k_T] g_{\text{hard}}(h_T^+) g_{\text{hard}}(h_T^-) \simeq 2g_{\text{hard}}(xq) \int [d^2 k_T] g_{\text{hard}}(k_T) \quad (41a)$$

$$\int [d^2 k_T] g_{\text{soft}}(h_T^+) g_{\text{hard}}(h_T^-) \simeq g_{\text{hard}}(xq) \int [dk_T^2] g_{\text{soft}}(k_T) \quad (41b)$$

in which the notation xq for the arguments of g refer to the

substitutions

$$\begin{aligned} k_{1T} &= (1 - x_1)q \\ k_{2T} &= -x_2q \\ k_{3T} &= -x_3q \end{aligned} \quad (41c)$$

which are the values of the transverse momenta assumed by one of the wave functions in the integrand when the other is at its peak (where its arguments h_T^\pm are zero). (Note that the approximations (41) hold only at high Q^2 when the integrands peak sharply at $h^\pm = 0$; they do not work for the product of two gaussians which do not peak sharply even at high Q^2 .) Using (41) and (37) the impulse approximation becomes

$$\begin{aligned} Q^4 G_{Mp}(Q^2) &= A^2 \int \left[\frac{Q^2}{2a^2} \right] \\ &+ B \, 24(16\pi^2)^2 \left\langle \frac{\phi^2(x)}{(1-x_1+x_3)^2 [(1-x_1+x_3)^2 + 3\epsilon x_2^2]} \right\rangle \end{aligned} \quad (42)$$

The A^2 term is usually referred to as the "soft contribution", and falls asymptotically like Q^{-2} . The term linear in B contains the effects of the high momentum tail of the wave function, and its asymptotic Q^2 dependence is a reflection of the high momentum behavior of this tail. Consistency with the hard calculation requires that B be chosen to reproduce the results (25). Using the form (35) for ϕ_s and $\epsilon = 1/16$,

$$\left\langle \frac{\phi^2}{(1-x_1+x_3)^2 [(1-x_1+x_3)^2 + 3\epsilon x_2^2]} \right\rangle \approx .00054 \, N_3^2 \quad (43)$$

If we choose a value for N_3 so that the PQCD calculation gives most (viz., two-thirds) of the experimental result, then the consistency condition gives the following value for B :

$$B = 1.24 \left(\frac{a_s}{\pi} \right)^2 = 1.03 \times 10^{-2} \quad (44)$$

if $a_s = 0.3$. Now the linear condition (37) gives

$$A = 1 - B I = 0.42 \quad (45)$$

for $\epsilon = 1/16$, $Q^2 = 10 \text{ GeV}^2$, and $\kappa = 300 \text{ MeV}$. Finally, the normalization condition (38) determines a new value of a

$$a \approx 0.24 \text{ GeV} \quad (46)$$

With this small value of Λ , the soft contribution to (42) is about 1/3 of the data level, even at its maximum value, which occurs at about $Q^2 \approx 10 (\text{GeV}/c)^2$.

Not only does the new wave function (36) show that the hard scattering calculation dominates for Q^2 of physical interest, it is a more realistic model wave function for the transverse momentum dependence of the proton. The term proportioned to B produces a tiny high k_T tail for the wave function which more accurately models the power law behavior of the proton wave function expected for large k_T . Adding such a behavior decouples the two conditions (37) and (39). It is not surprising that this small "tail" (a) contributes the major part of the strength required for the hard scattering result, (b) dominates the impulse approximation at large Q^2 [if the wave function were exact, the extra term in (42) should reproduce the result (25) exactly], and (c) that it plays no role in the normalization of the wave function, which is dominated by small momentum components. Furthermore, the tail is so small that it also plays no role in the RMS value of k_T .

The first term in the sum (36) is the only term which contains truly small values of k_T , and the contribution of this term to the impulse approximation is the "soft contribution" referred to above. By moving some of the strength in Eq. (37) to the tail, the soft contribution has been reduced by a factor of about nine and no longer dominates the form factor.

Summarizing, consideration of the tail has resulted in

- a reduction of the soft contributions to G_{Mp} by a factor of about nine, so that at their peak they are below the data, and
- decoupling of the normalization condition (a non-perturbative low momentum effect) from the asymptotic calculation (a high momentum effect).

Thus, the soft contributions do not necessarily dominate the PQCD contributions to the form factor.

IV. COMMENTS AND CONCLUSIONS

In conclusion, the following comments are offered.

1. Factored wave function. For simplicity, a factored form for the wave function has been used in this paper. This is probably an oversimplification; the correct wave function is not likely to be factorizable. The hard and soft regions of transverse momenta could easily have different x -dependencies, and this can give us significant extra freedom to manipulate the hard and soft contributions.

2. Asymptotic ratio G_{Mp}/G_{Mn} . It has been noted¹ that at very, very high Q^2 (that is, $\ln \ln Q^2 \gg 1$) only the zeroth Appell polynomial survives and the proton form factor goes to zero relative to the neutron form factor. It should also be noted that in this limit the neutron form factor is positive, so that the neutron form factor must have a zero¹⁴ at some large but finite Q^2 .

3. Chernyak and Zhitnitsky distribution amplitude.⁵ Chernyak and Zhitnitsky have proposed a distribution amplitude for the proton. Their distribution amplitude is gotten by supposing an expansion in terms of the six lowest Appell polynomials and fitting to six moments that are calculated using QCD sum rules, and is

$$\begin{aligned} \phi(x) = & x_1 x_2 x_3 (0.111\tilde{\phi}_0 - 0.274\tilde{\phi}_1 - 0.212\tilde{\phi}_2 \\ & + 0.248\tilde{\phi}_3 + 0.221\tilde{\phi}_4 + 0.002\tilde{\phi}_5) \text{ GeV}^2. \end{aligned}$$

This distribution amplitude gives a good account of G_{Mp} , gives $G_{Mn} \approx -(1/2)G_{Mp}$, and is quite asymmetric. While this distribution amplitude is not uniquely forced by the calculated moments, those moments do not allow the possibility of no asymmetry.

As an amusement, examine the hard scattering expression for G_{Mp} and note that every single term there is positive if N_0 , N_3 , and N_4 have one sign and N_1 , N_2 , and N_5 have the opposite sign. This is just the sign pattern in the Chernyak-Zhitnitsky distribution amplitude, excepting the last term whose coefficient is too small to be

significant. The QCD sum rules have thus led to a distribution amplitude which satisfies one clear criterion for maximizing G_{M_p} .

4. Asymmetric wave function. The sorts of wave functions that fit both the neutron and proton form factors are quite asymmetric in the three quarks. This is perhaps a surprise and it may be worth speculating how it may come about. First, note that the distribution amplitude, which is a transverse momentum integrated wave function, is dominated by the high k_T part of the wave function (if the wave function falls as a power of k_T as expected from PQCD). At the same time, the normalization (which unlike the distribution amplitude is gotten by squaring the wave function before integrating) is dominated by low k_T . The expectation of near symmetry among the quarks comes from calculations of things like the charge radius or magnetic moment that are like the normalization in being dominated by the low k_T part of the wave function, and the x -dependence associated with this could be quite symmetric.

Why, then, might one expect an asymmetry at high k_T ? Think of quark-quark scattering, or equally well, electron-electron scattering at very high energies. There is a large, angle-dependent spin dependence. At 90° in the c.m., the amplitude for scattering two same helicity electrons is twice the magnitude of the amplitude for opposite helicity electrons. High k_T quarks result from a hard scattering of low k_T quarks, and this amplitude is spin dependent. The pair of quarks with same helicity are more likely to scatter each other out to high k_T than other pairs of quarks, and this same scattering will likely also scatter the quarks forward and backward so that one of the same helicity quarks will have a large share of the longitudinal momentum. This is just what is seen.

To conclude: the magnitude and sign of either nucleon magnetic form factor can be fit with a broad distribution amplitude, and consideration of both nucleons together suggests an asymmetric spatial part of the distribution amplitude. Finally, the soft contributions may be below the asymptotic QCD results in the range where experiments may support the latter.

Acknowledgments

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13. The eventual result does however involve a broad wave function which means that G_M gets much of its size from contributions close to the kinematic boundaries. This could make the result sensitive to inserting quark masses or effective gluon masses.
14. A preliminary account of the present work was reported by C. E. Carlson, in Nucleon and Nuclear Structure and Exclusive Electromagnetic Interaction Studies, Proceedings of the Third Workshop of the Bates Users Theory Group, 23-24 July 1984, G. H. Rawitscher, ed., University of Connecticut, 1984.
15. Parts of these formulas were obtained by Andrikopoulou, Ref. 6. We don't agree with all the terms there, however.

- Figure 1. Data for $Q^4 G_{Mp}$ plotted vs. Q^2 . (Taken from Ref. 8.) The two dashed lines indicate how the data would behave if $G_{Mp} \sim 1/Q^2$ or $G_{Mp} \sim 1/Q^6$.
- Figure 2. The process giving G_{Mp} . Three parallel moving quarks enter the circle labelled T_H where one of them absorbs the photon entering mainly from a transverse direction, and then shares the momentum with its fellows so that three parallel quarks emerge.
- Figure 3. Lowest order perturbation diagrams for T_H . The small +/- signs indicate quark helicities.
- Figure 4. G_{Mp}/G_{Mn} for the simple symmetric wave function, plotted vs. the power parameter for η .
- Figure 5. The impulse approximation which generates both hard and soft contributions to the form factor.
- Figure 6. The function $f(\xi)$ defined in Eq. (34).

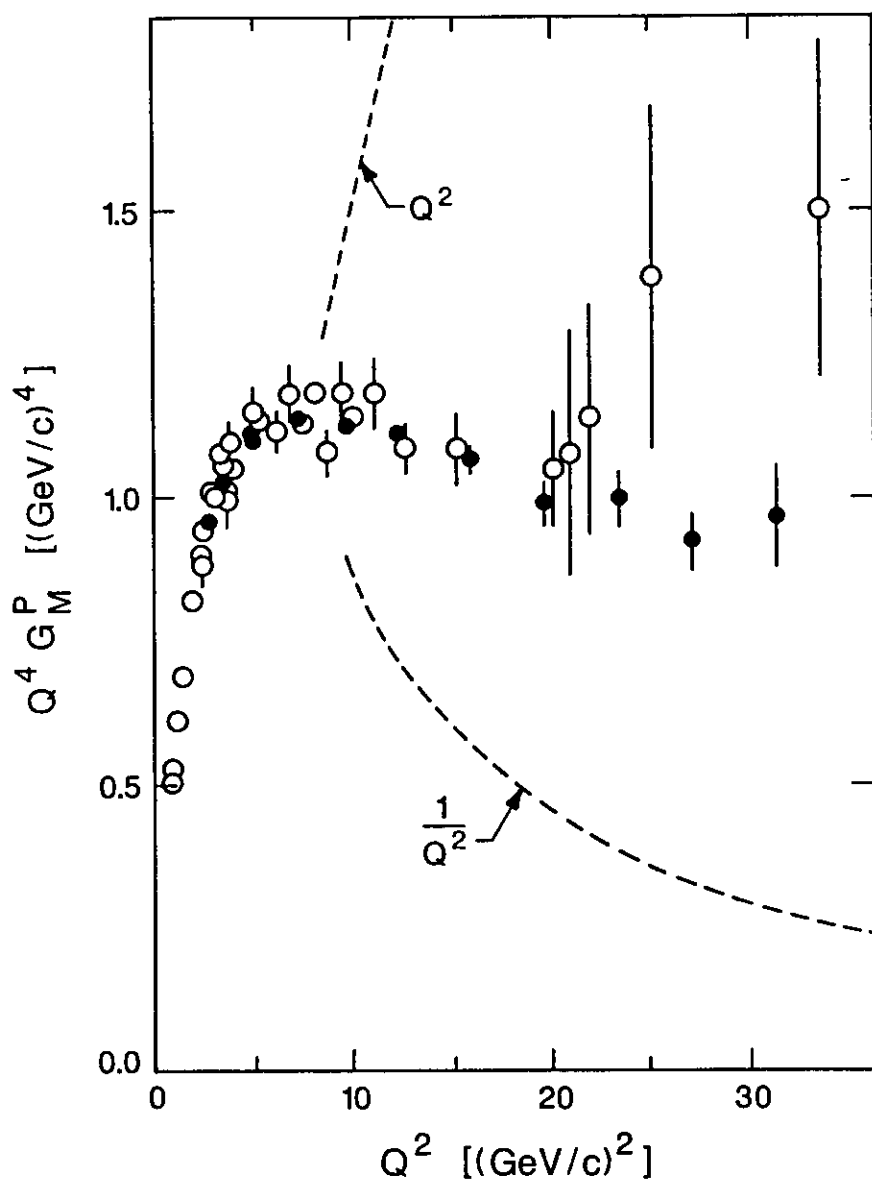


Figure 1

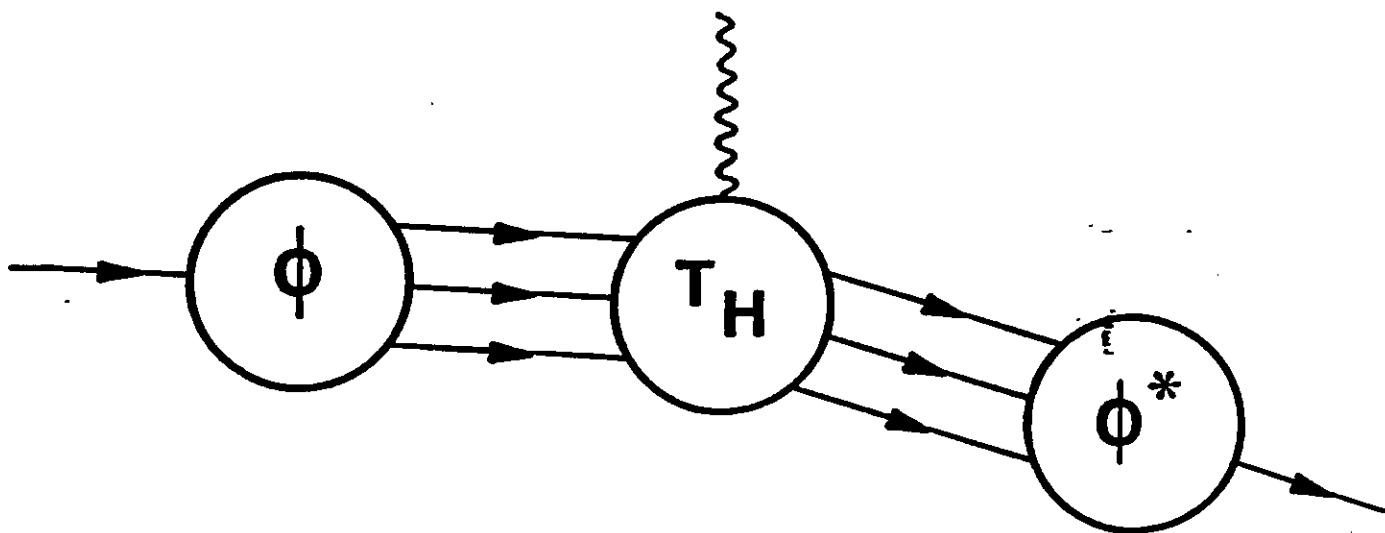


Figure 2

$$\begin{aligned}
 T_H = & \begin{array}{c} + \\ - \\ + \end{array} \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \text{wavy} \\ \text{---} \bullet \text{---} \text{wavy} \text{---} \bullet \text{---} \text{wavy} \text{---} \bullet \text{---} \text{wavy} \\ \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \text{wavy} \end{array} + \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \text{wavy} \\ \text{---} \text{wavy} \text{---} \bullet \text{---} \bullet \text{---} \text{wavy} \\ \text{---} \bullet \text{---} \bullet \text{---} \text{wavy} \end{array} + \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \text{wavy} \\ \text{---} \bullet \text{---} \text{wavy} \text{---} \bullet \text{---} \text{wavy} \\ \text{---} \text{wavy} \text{---} \bullet \text{---} \bullet \text{---} \text{wavy} \end{array} + \\
 & \begin{array}{c} + \\ - \\ + \end{array} \begin{array}{c} \text{---} \text{wavy} \text{---} \bullet \text{---} \bullet \text{---} \text{wavy} \\ \text{---} \bullet \text{---} \text{wavy} \text{---} \bullet \text{---} \text{wavy} \\ \text{---} \text{wavy} \text{---} \bullet \text{---} \bullet \text{---} \text{wavy} \end{array} + (\text{IN.} \leftrightarrow \text{OUT.})
 \end{aligned}$$

Figure 3

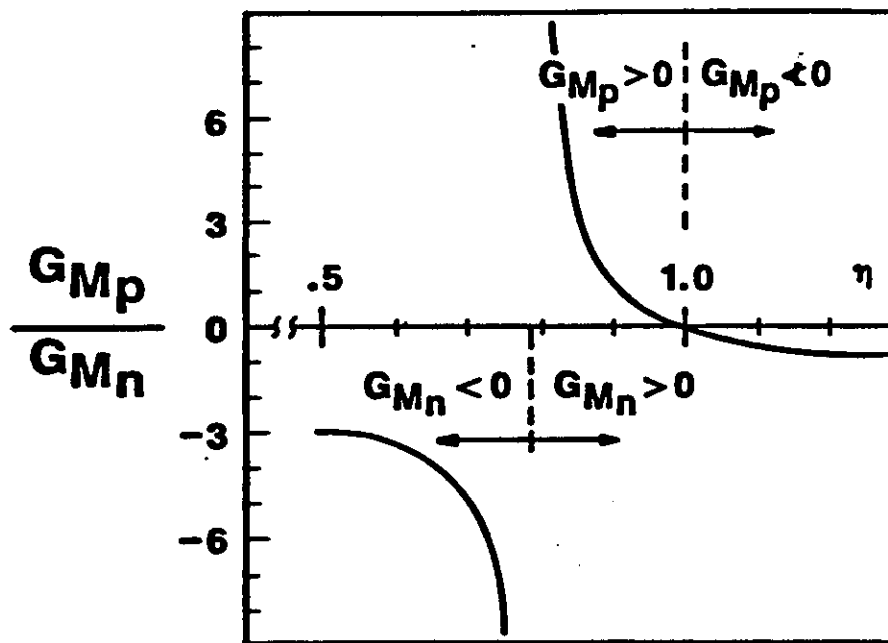


Figure 4

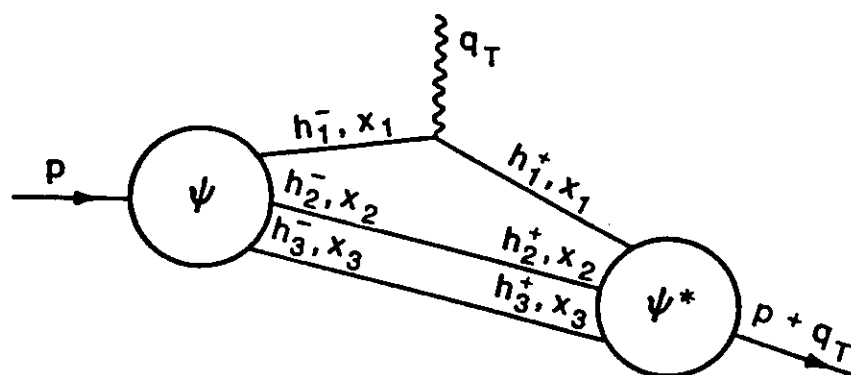


Figure 5

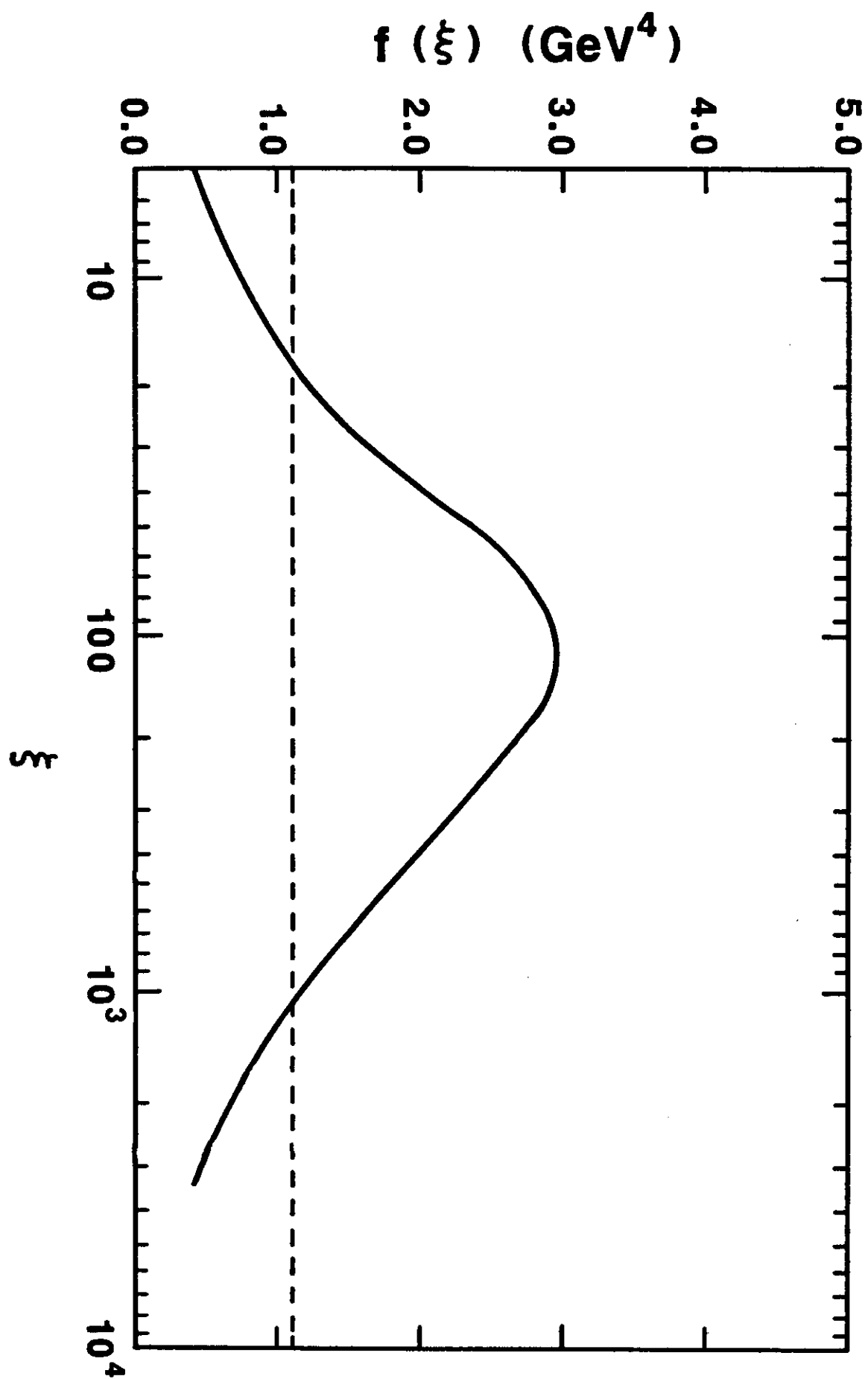


Figure 6

